An Application of Statistical Thermodynamics to an Open Flow System

Chukwuneke Jeremiah Lekwuwa, Achebe Chinonso Hubert, Okolie Paul Chukwulozie, Ajike Chinagorom Ogbu

Department of Mechanical Engineering,
Nnamdi Azikiwe University, PMB 5025, Awka, Nigeria

Correspondance Email: jl.chukwuneke@unizik.edu.ng

Abstract—This research work, an application of statistical (analytical) thermodynamics to an Open flow system used the grand canonical ensemble to deal with the study of a liquid flow system. The essence was to understand and to interpret measurable macroscopic properties of materials and thus establish valid relationships among measureable variables in a statistical manner over time. A valid relationship with constant, k_e between the energy (E) supplied to 2hp pump and the expected discharge (Q) at a 50mm riser was established. This thus reiterates the relationship among important variables for optimal outputs and longevity of the system. Absolute values of energy and the discharge quantities were obtained E_{abs} = 1305.5122 Joules and Q_{abs} = 0.0095m^3/s respectively.

Index Terms— Analytical Thermodynamics, Bernoulli Equation, Boltzmann Factors, Discharge, Energy, Grand Canonical Ensemble, Macroscopic Properties, Quantum Mechanics, Statistical Thermodynamics.

1. INTRODUCTION

The essential task in this application of statistical thermodynamics is to determine the distribution of a given amount of energy E over N identical systems. The goal is to understand and to interpret the measurable macroscopic properties of materials in terms of the properties of their constituent particles and the interactions between them. This is done by connecting thermodynamic function to quantum-mechanic equations. Two central quantities in statistical thermodynamics are the Boltzmann factor and the partition factor.

2. AIM OF THE RESEARCH

The aim of this research is to statistically collate data of Power (P) developed by the pump, Energy (E) supplied to the fluid by the pump, Discharge (Q) at the riser over different ranges of Voltages (V) available to the pump.

Obtaining these values and applying them to the grand canonical ensemble is intended to ultimately establish a relationship between the Energy (E) supplied to the fluid by the pump and the subsequent discharge (Q) at the riser with a possible constant k, thus defining the relationship at any specified value.

3. ADOPTION OF REAL SYSTEM

The real system adopted for the purpose of this experimental analysis is an integral part of the Water Treatment plant of the Federal Ministry of Works Headquarters, Abuja Nigeria.

This research work adopted a system of 2hp pump to transport water from the ground tank to an overhead tank. The capacity of the ground tank is 10,000 litres while the overhead tank is 30,000 litres. The two tanks were connected with a riser of 50mm diameter pressurized PVC pipe.

The Headquarter Complex is a three story building with a floor-to-floor distance of 3.5meters. This gives the height of the building from the ground (i.e. height of water to be transported) to be 3.5 meters X 4 floors (including the

- *Chukwuneke J.L is currently pursuing doctorate degree program in mechanical engineering in Nnamdi AzikiweUniversity, Awka, Nigeria, PH-+2348033574353. E-mail: jl.chukwuneke@unizik.edu.ng*
- *Achebe C.H is currently a senior Lecturer in mechanical engineering in Nnamdi Azikiwe University, Awka, Nigeria.*
- *Okolie P.C is currently pursuing doctorate degree program in mechanical engineering in Nnamdi Azikiwe University, Awka, Nigeria.*
- *Ajike C.O is currently pursuing masters degree program in mechanical engineering in Nnamdi Azikiwe University, Akwa,Nigeria*

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ground floor). Therefore, the height of water to be transported is 14 meters.

4. GOVERNING EQUATION

The governing equation of the system is the Bernoulli equation which states that "the sum of energy at different points along a cross-section is constant." Mathematically:

$$\left[p + \frac{v^2}{\rho} + gZ\right]_{out} = \left[p + \frac{v^2}{\rho} + gZ\right]_{in} - \text{losses} - w_s \qquad (1)$$

Where, P = pressure of the fluid, ϱ = density of the fluid, V = velocity of the fluid, z = height from datum, g = acceleration due to gravity, w_s = shaft work done by the fluid per unit mass flowing.

Another important equation is the one showing the relationship between Power (P), Discharge (Q), density (ϱ), height (H) and the acceleration due to gravity (g).

$$P = Q.\,\varrho.g.\,H \qquad (2)$$

5. BASIC ASSUMPTION

If we consider the energy supplied to the fluid per unit time, then Energy (E) supplied to the fluid can be given as:

$$\text{Energy} = (Q.\,\varrho.g.\,H) \text{ in joule} \qquad (3)$$

Where, E = energy supplied to the fluid (joule), Q= discharge (m^3/s), ϱ = density of the fluid (kg/m^3), H = Piezometric height of water (meters).

6. PRESENTATION OF DATA

Referring to Eqn. (3) And considering the data obtained from the Water Treatment Plant, as presented in table 1.

TABLE 1
Showing Data For Power Developed And Voltage Supplies [7]

S/N	Voltage (Volts)	Power (hp)	Power (Watt)
1	240.00	2.00	1492.00
2	238.00	2.00	1492.00
3	236.00	2.00	1492.00
4	234.00	1.90	1417.40
5	232.00	1.90	1417.40
6	230.00	1.90	1417.40
7	228.00	1.80	1342.80
8	226.00	1.75	1305.50
9	224.00	1.75	1305.50
10	222.00	1.70	1305.50
11	220.00	1.70	1268.20
12	218.00	1.60	1193.60
13	216.00	1.60	1193.60
14	214.00	1.50	1119.00
15	212.00	1.50	1119.00
16	210.00	1.50	1119.00
17	208.00	0.00	0.0000

It could be noted that at maximum allowable voltage supply (240volts), the power developed by the pump is 2hp. As the voltage supply is reduced, the power developed by the pump is also reduced until at 208volts the power developed fell to zero. This shows that as the voltage supply reduced below 210volts, the pump could not be actuated hence, no power could be developed as shown on table 1. This relationship is also shown on fig. 1 below.

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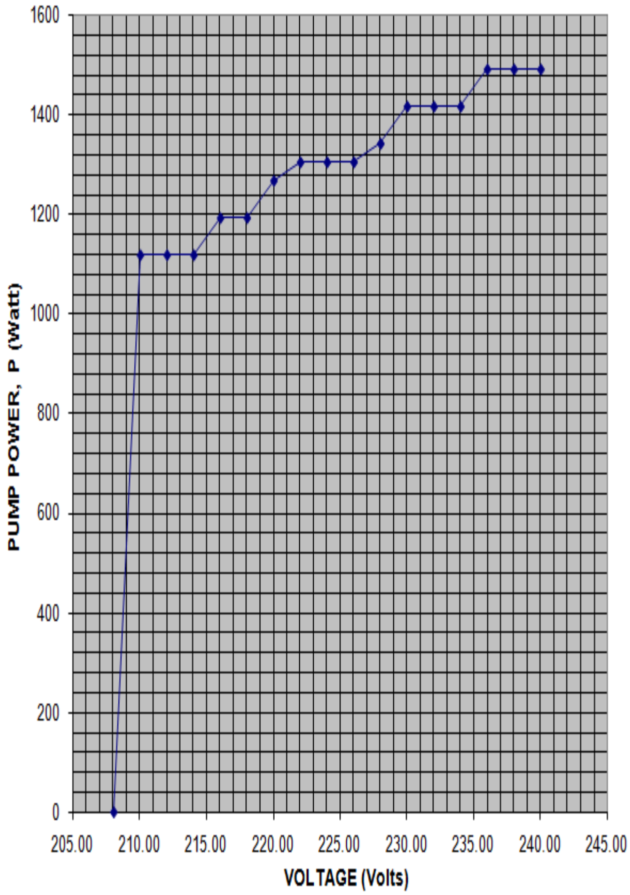

Fig.1: Graph of Pump Power, P (Watt) Against Voltage, V (Volts)

It could be recalled (from basic assumption) that the time is taken to be unity hence, the numerical value of Power developed by the pump is equal to the Energy supplied to the fluid. The value of the Discharge Q (m^3/s), at different values of the voltages (V) supplied to the pump was determined using Eqn. (3).

TABLE 2

Showing Data For Discharge (Q) And Energy (E)

S/N	Voltage (Volts)	Discharge (m^3/s)	Energy (Joule)
1	240.00	0.0109	1492.00
2	238.00	0.0109	1492.00
3	236.00	0.0109	1492.00
4	234.00	0.0103	1417.40
5	232.00	0.0103	1417.40
6	230.00	0.0103	1417.40
7	228.00	0.0098	1342.80
8	226.00	0.0095	1305.50
9	224.00	0.0095	1305.50
10	222.00	0.0092	1305.50
11	220.00	0.0092	1268.20
12	218.00	0.0087	1193.60
13	216.00	0.0087	1193.60
14	214.00	0.0081	1119.00
15	212.00	0.0081	1119.00
16	210.00	0.0081	1119.00
17	208.00	0.00	0.0000

- The relationship between Discharge (Q) and the voltage (V) is shown on Figure 2.
- The relationship between Energy (E) and the voltage (V) is shown on Figure 3.
- The relationship between Discharge (Q) and the Energy (E) is shown on Figure 4.

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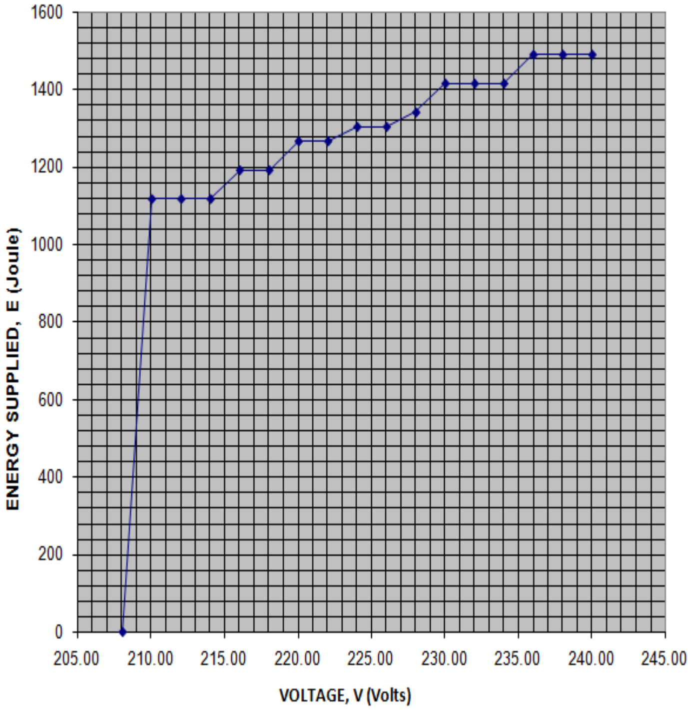

Fig 2: Graph of Flow Discharge, Q (M^3/S) Against Voltage (Volts)

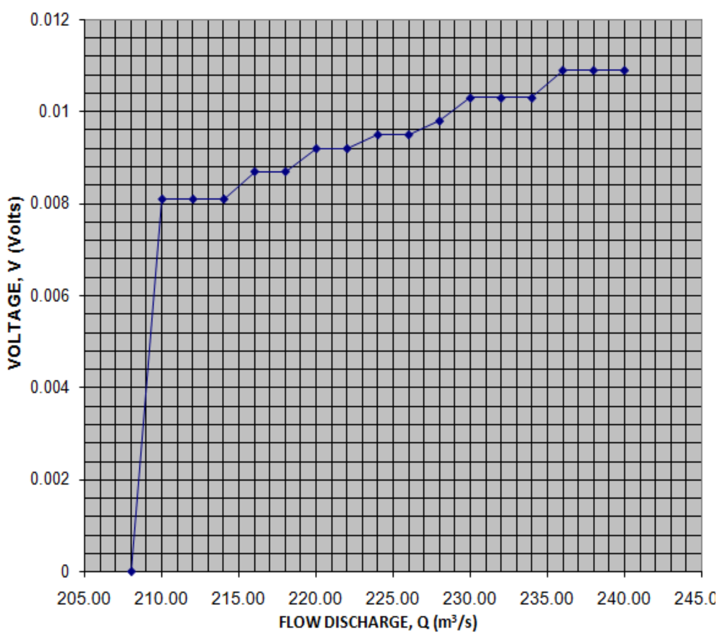

Fig 3: Graph of Energy Supplied By Pump, E (Joule) Against Voltage, V (Volts)

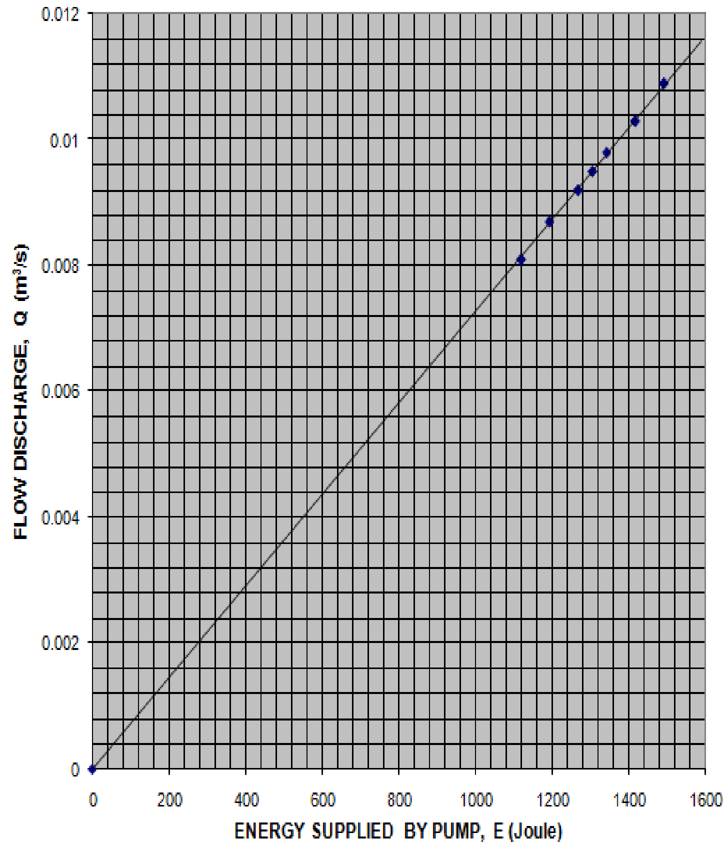

Fig 4: Graph of Flow Discharge (M^3/S) Against Energy Supplied By Pump, E (Joule)

A numerical computing environment MATLAB (*matrix laboratory*) was used to obtain the mean (absolute) value of discharge Q_{abs} and the energy E_{abs} supplied to the fluid. These absolute values are as stated below;

$Q_{abs} = 0.009\,5\ m^3/s$

$E_{abs} = 1305.512\,2$ Joule

7. CALCULATIONS AND FORMULATION OF RELATIONSHIPS

The relationship between the measured and calculated variables is best done from graphs. For the purpose of this research work, the results obtained for the discharge Q and the Energy E are represented on a log – log graph in fig. 5.

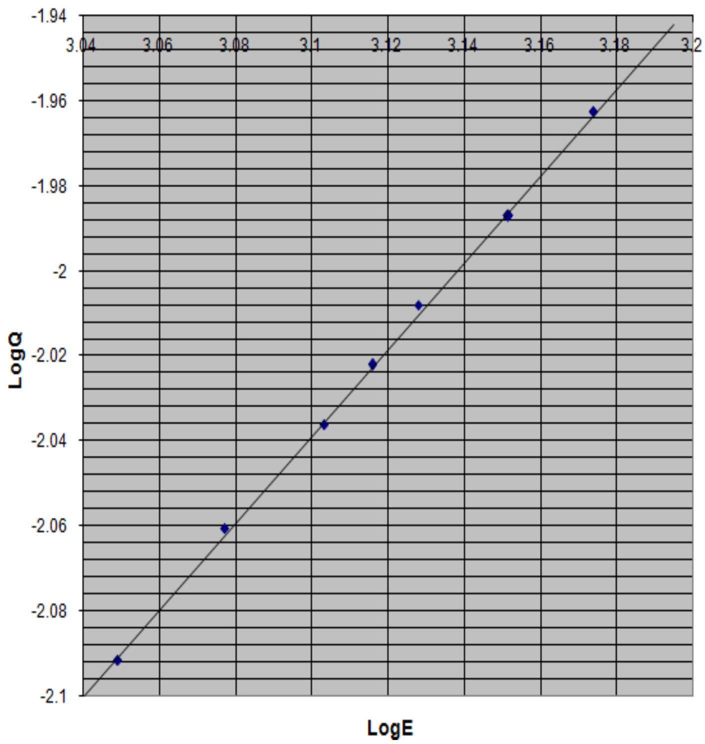

Fig 5: Graph of LogQ against LogE

From elementary arithmetic, it could be understood that the equation of a straight line is given as; y = mx + c

Where, m = slope of the graph, c = intercept on the log Q axis, y represents the Log Q axis, x represents the Log E axis.

From Fig. 5, the slope is calculated as:

Slope = $\frac{\Delta logQ\ axis}{\Delta logE\ axis} = \frac{-1.96-(-2.08)}{3.17-3.06} = 0.2350$

Therefore, applying the equation of a straight line the equation becomes: Log Q = 0.235Log E + (-2.1000)

Or

Log Q = (0.235Log E) -2.1000 (4)

Equation 4, above has been established as a valid relationship between the amount of energy (E), supplied to the fluid and the discharge (Q) at the riser.

8. RELATING TO GRAND CANONICAL ENSEMBLE

The system described above is an open system or a control volume with exchange of matter. Since the second law of thermodynamics applies to open systems, the Grand canonical ensemble also describes an open system. In Grand canonical ensemble μ, V and T are fixed [4].

Since an open system does not keep a constant energy, the total amount of energy in the system will fluctuate. Thus, the system can access only those of its macro-state that corresponds to a given value of E of the energy. The equation for the grand canonical ensemble can be represented as [5]:

$PV = K_B T In (\Omega(E))$ (5)

From the foregoing analysis, an equation of the form in eqn (1.4), using the values of Q_{abs} and E_{abs} could be obtained.

Since V and T are constant, and assumed unity. Representing PV as the absolute Discharge (Q_{abs}) and In (Ω(E)) as the absolute Energy, (E_{abs}) supplied to the fluid.

Therefore;

$Q_{abs} = K_c E_{abs}$ (6)

Where, k_c describes the constant that relates Q and E.

It has been shown on Figure 4, that Q is directly proportional to E. Since there is a variation, a constant of proportionality must define the relationship. This proportionality constant, k_c is gotten from the application of Thermodynamics Ensemble as seen in Eqn. (6).

From Eqn. (6), $k_c = \frac{Q_{abs}}{E_{abs}}$

$k_c = \frac{0.00953201}{1305.51221} = 7.3 \times 10^{-6}\ m^3/Js$ (constant of proportionality)

It could be noted that all relations established in this research work are valid only for voltages between 210 to 240 volts supplied to the pump. This is so because this is the range at which the functionality of the pump can be guaranteed.

9. CONCLUSION

This research work has shown the interface between statistical thermodynamics and real system using grand canonical ensemble. It has helped to establish a valid relationship and a constant, k_c between the energy (E) supplied to a 2hp pump and the expected discharge (Q) at the 50mm riser. This modern research techniques and challenges abound in research work.

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